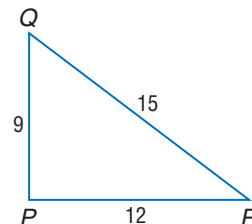


## Example 2 Geometric Conjecture

For points  $P$ ,  $Q$ , and  $R$ ,  $PQ = 9$ ,  $QR = 15$ , and  $PR = 12$ . Make a conjecture and draw a figure to illustrate your conjecture.

**Given:** points  $P$ ,  $Q$ , and  $R$ ;  $PQ = 9$ ,  $QR = 15$ , and  $PR = 12$   
 Examine the measures of the segments.  
 Since  $PQ + PR \neq QR$ , the points cannot be collinear.

**Conjecture:**  $P$ ,  $Q$ , and  $R$  are noncollinear.



**FIND COUNTEREXAMPLES** A conjecture based on several observations may be true in most circumstances, but false in others. It takes only one false example to show that a conjecture is not true. The false example is called a **counterexample**.

## Example 3 Find a Counterexample

**FINANCE** Find a counterexample for the following statement based on the graph.

*The rates for CDs are at least 1.5% less than the rates a year ago.*

Examine the graph. The statement is true for 6-month, 1-year, and  $2\frac{1}{2}$ -year CDs. However, the difference in the rate for a 5-year CD is 0.74% less, which is less than 1.5%. The statement is false for a 5-year certificate of deposit. Thus, the change in the 5-year rate is a counterexample to the original statement.



**Log on for:**

- Updated data
- More on finding counterexamples

[www.geometryonline.com/usa\\_today](http://www.geometryonline.com/usa_today)



### USA TODAY Snapshots®

#### Latest CD rates

Average certificate of deposit rates as of Wednesday:

<b>6-month</b>	This week	1.80%
	Last week	1.80%
	Year ago	4.55%
<b>1-year</b>	This week	2.12%
	Last week	2.11%
	Year ago	4.64%
<b>2½-year</b>	This week	2.96%
	Last week	2.96%
	Year ago	4.74%
<b>5-year</b>	This week	4.22%
	Last week	4.23%
	Year ago	4.96%

Source: Bank Rate Monitor, 800-327-7717, [www.bankrate.com](http://www.bankrate.com)

USA TODAY

## Check for Understanding

- Concept Check**
1. Write an example of a conjecture you have made outside of school.
  2. Determine whether the following conjecture is *always*, *sometimes*, or *never* true based on the given information.  
**Given:** collinear points  $D$ ,  $E$ , and  $F$   
**Conjecture:**  $DE + EF = DF$
  3. **OPEN ENDED** Write a statement. Then find a counterexample for the statement.



**Guided Practice** Make a conjecture about the next item in each sequence.

4. 
5.  $-8, -5, -2, 1, 4$

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

6.  $PQ = RS$  and  $RS = TU$                       7.  $\overline{AB}$  and  $\overline{CD}$  intersect at  $P$ .

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

8. **Given:**  $x$  is an integer.  
**Conjecture:**  $-x$  is negative.
9. **Given:**  $WXYZ$  is a rectangle.  
**Conjecture:**  $WX = YZ$  and  $WZ = XY$

**Application** 10. **HOUSES** Most homes in the northern United States have roofs made with steep angles. In the warmer areas of the southern states, homes often have flat roofs. Make a conjecture about why the roofs are different.

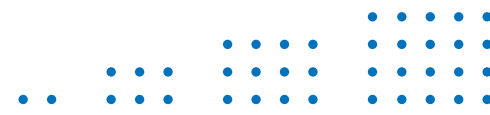

## Practice and Apply

### Homework Help

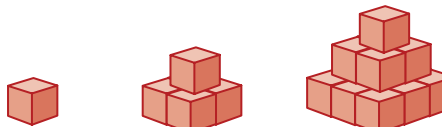
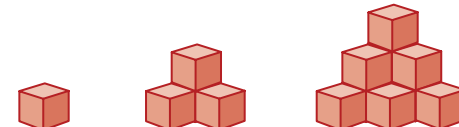
For Exercises	See Examples
11–20	1
21–28	2
29–36	3

**Extra Practice**  
See page 756.

Make a conjecture about the next item in each sequence.

11. 
12. 
13.  $1, 2, 4, 8, 16$                       14.  $4, 6, 9, 13, 18$                       15.  $\frac{1}{3}, 1, \frac{5}{3}, \frac{7}{3}, 3$
16.  $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$                       17.  $2, -6, 18, -54$                       18.  $-5, 25, -125, 625$

Make a conjecture about the number of blocks in the next item of each sequence.

19.                       20. 

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

21. Lines  $\ell$  and  $m$  are perpendicular.                      22.  $A(-2, -11), B(2, 1), C(5, 10)$
23.  $\angle 3$  and  $\angle 4$  are a linear pair.                      24.  $\overline{BD}$  is an angle bisector of  $\angle ABC$ .
25.  $P(-1, 7), Q(6, -2), R(6, 5)$                       26.  $HJK$  is a square.
27.  $PQRS$  is a rectangle.                      28.  $\angle B$  is a right angle in  $\triangle ABC$ .

## WebQuest

You can use scatter plots to make conjectures about the relationships between latitude, longitude, degree distance, and the monthly high temperature. Visit [www.geometryonline.com/WebQuest](http://www.geometryonline.com/WebQuest) to continue work on your WebQuest project.

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

29. **Given:**  $\angle 1$  and  $\angle 2$  are complementary angles.

**Conjecture:**  $\angle 1$  and  $\angle 2$  form a right angle.

30. **Given:**  $m + y \geq 10$ ,  $y \geq 4$

**Conjecture:**  $m \leq 6$

31. **Given:** points  $W$ ,  $X$ ,  $Y$ , and  $Z$

**Conjecture:**  $W$ ,  $X$ ,  $Y$ , and  $Z$  are noncollinear.

32. **Given:**  $A(-4, 8)$ ,  $B(3, 8)$ ,  $C(3, 5)$

**Conjecture:**  $\triangle ABC$  is a right triangle.

33. **Given:**  $n$  is a real number.

**Conjecture:**  $n^2$  is a nonnegative number.

34. **Given:**  $DE = EF$

**Conjecture:**  $E$  is the midpoint of  $\overline{DF}$ .

35. **Given:**  $JK = KL = LM = MJ$

**Conjecture:**  $JKLM$  forms a square.

36. **Given:** noncollinear points  $R$ ,  $S$ , and  $T$

**Conjecture:**  $\overline{RS}$ ,  $\overline{ST}$ , and  $\overline{RT}$  form a triangle.

- 37. **MUSIC** Many people learn to play the piano by ear. This means that they first learned how to play without reading music. What process did they use?

**CHEMISTRY** For Exercises 38–40, use the following information.

Hydrocarbons are molecules composed of only carbon (C) and hydrogen (H) atoms. The simplest hydrocarbons are called alkanes. The first three alkanes are shown below.

Alkanes			
Compound Name	Methane	Ethane	Propane
Chemical Formula	$\text{CH}_4$	$\text{C}_2\text{H}_6$	$\text{C}_3\text{H}_8$
Structural Formula	$\begin{array}{c} \text{H} \\   \\ \text{H}-\text{C}-\text{H} \\   \\ \text{H} \end{array}$	$\begin{array}{c} \text{H} \quad \text{H} \\   \quad   \\ \text{H}-\text{C}-\text{C}-\text{H} \\   \quad   \\ \text{H} \quad \text{H} \end{array}$	$\begin{array}{c} \text{H} \quad \text{H} \quad \text{H} \\   \quad   \quad   \\ \text{H}-\text{C}-\text{C}-\text{C}-\text{H} \\   \quad   \quad   \\ \text{H} \quad \text{H} \quad \text{H} \end{array}$

38. Make a conjecture about butane, which is the next compound in the group. Write its structural formula.
39. Write the chemical formula for the 7th compound in the group.
40. Develop a rule you could use to find the chemical formula of the  $n$ th substance in the alkane group.
41. **CRITICAL THINKING** The expression  $n^2 - n + 41$  has a prime value for  $n = 1$ ,  $n = 2$ , and  $n = 3$ . Based on this pattern, you might conjecture that this expression always generates a prime number for any positive integral value of  $n$ . Try different values of  $n$  to test the conjecture. Answer *true* if you think the conjecture is always true. Answer *false* and give a counterexample if you think the conjecture is false.

## More About . . .



### Music

The average medium-sized piano has about 230 strings. Each string has about 165 pounds of tension. That's a combined tension of about 18 tons.

Source: [www.pianoworld.com](http://www.pianoworld.com)

42. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How can inductive reasoning help predict weather conditions?**

Include the following in your answer:

- an explanation as to how a conjecture about a weather pattern in the summer might be different from a similar weather pattern in the winter, and
- a conjecture about tomorrow's weather based on your local weather over the past several days.

EOC Practice

Standardized  
Test Practice

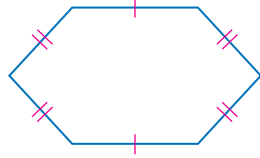
43. What is the next term in the sequence 1, 1, 2, 3, 5, 8?  
 (A) 11 (B) 12 (C) 13 (D) 14
44. **ALGEBRA** If the average of six numbers is 18 and the average of three of the numbers is 15, then what is the sum of the remaining three numbers?  
 (A) 21 (B) 45 (C) 53 (D) 63

## Maintain Your Skills

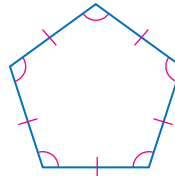
### Mixed Review

Name each polygon by its number of sides and then classify it as *convex* or *concave* and *regular* or *irregular*. (Lesson 1-6)

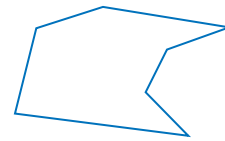
45.



46.

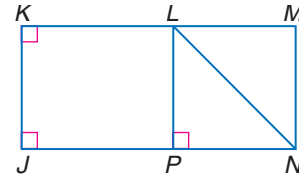


47.



Determine whether each statement can be assumed from the figure. Explain. (Lesson 1-5)

48.  $\angle KJN$  is a right angle.  
 49.  $\angle PLN \cong \angle NLM$   
 50.  $\angle PNL$  and  $\angle MNL$  are complementary.  
 51.  $\angle KLN$  and  $\angle MLN$  are supplementary.  
 52.  $\angle KLP$  is a right angle.



Find the coordinates of the midpoint of a segment having the given endpoints. (Lesson 1-3)

53.  $\overline{AB}$  for  $A(-1, 3)$ ,  $B(5, -5)$   
 54.  $\overline{CD}$  for  $C(4, 1)$ ,  $D(-3, 7)$   
 55.  $\overline{FG}$  for  $F(4, -9)$ ,  $G(-2, -15)$   
 56.  $\overline{HJ}$  for  $H(-5, -2)$ ,  $J(7, 4)$   
 57.  $\overline{KL}$  for  $K(8, -1.8)$ ,  $L(3, 6.2)$   
 58.  $\overline{MN}$  for  $M(-1.5, -6)$ ,  $N(-4, 3)$

Find the value of the variable and  $MP$ , if  $P$  is between  $M$  and  $N$ . (Lesson 1-2)

59.  $MP = 7x$ ,  $PN = 3x$ ,  $MN = 24$   
 60.  $MP = 2c$ ,  $PN = 9c$ ,  $MN = 63$   
 61.  $MP = 4x$ ,  $PN = 5x$ ,  $MN = 36$   
 62.  $MP = 6q$ ,  $PN = 6q$ ,  $MN = 60$   
 63.  $MP = 4y + 3$ ,  $PN = 2y$ ,  $MN = 63$   
 64.  $MP = 2b - 7$ ,  $PN = 8b$ ,  $MN = 43$

### Getting Ready for the Next Lesson

**BASIC SKILL** Determine which values in the given replacement set make the inequality true.

65.  $x + 2 > 5$   
 {2, 3, 4, 5}
66.  $12 - x < 0$   
 {11, 12, 13, 14}
67.  $5x + 1 > 25$   
 {4, 5, 6, 7}