

## Section 7: Systems of Equations

In this section, we are going to learn skills for:

### *NGSS Standards*

- ☀ **MA.912.A.3.13** Use a graph to approximate the solution of a system of linear equations or inequalities in two variables with and without technology. (Assessed with MA.912.A.3.14.)
- ☀ **MA.912.A.3.14** Solve systems of linear equations and inequalities in two and three variables using graphical, substitution, and elimination methods. (Also assesses MA.912.A.3.13 and MA.912.A.3.15.)
- ☀ **MA.912.A.3.15** Solve real-world problems involving systems of linear equations and inequalities in two and three variables. (Assessed with MA.912.A.3.14.)

### *CCS Standards*

- ☉ **MACC.912.A-REI.3.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- ☉ **MACC.912.A-REI.3.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

## Section 7 – Video 1

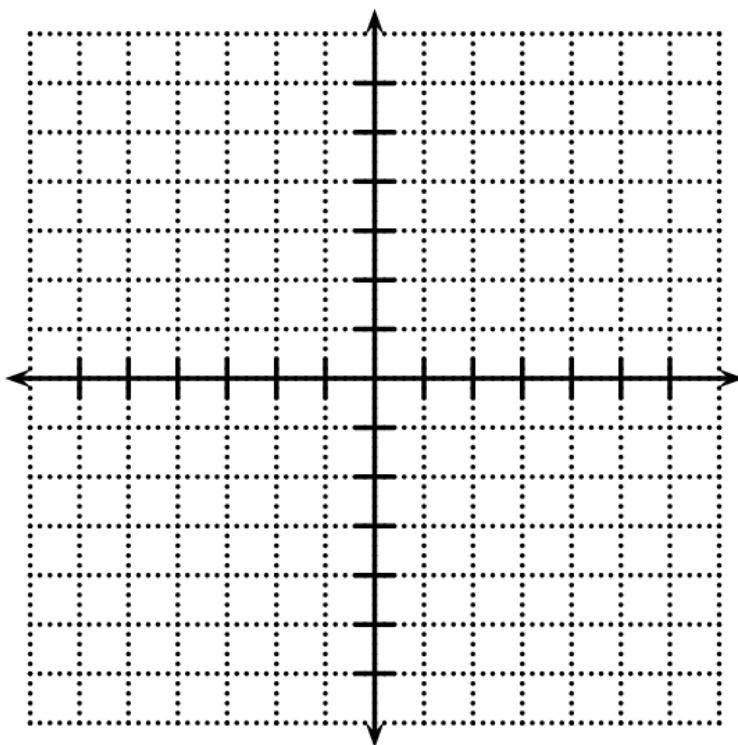
### What is a System of Linear Equations?

A **system of linear equations** is a set of two or more lines. For our purposes, we will only be considering two lines.

- We are given two equations of lines that use the same variables.
- Our goal is to find where the two lines intersect.

$$y = x - 4$$
$$y = -x + 2$$

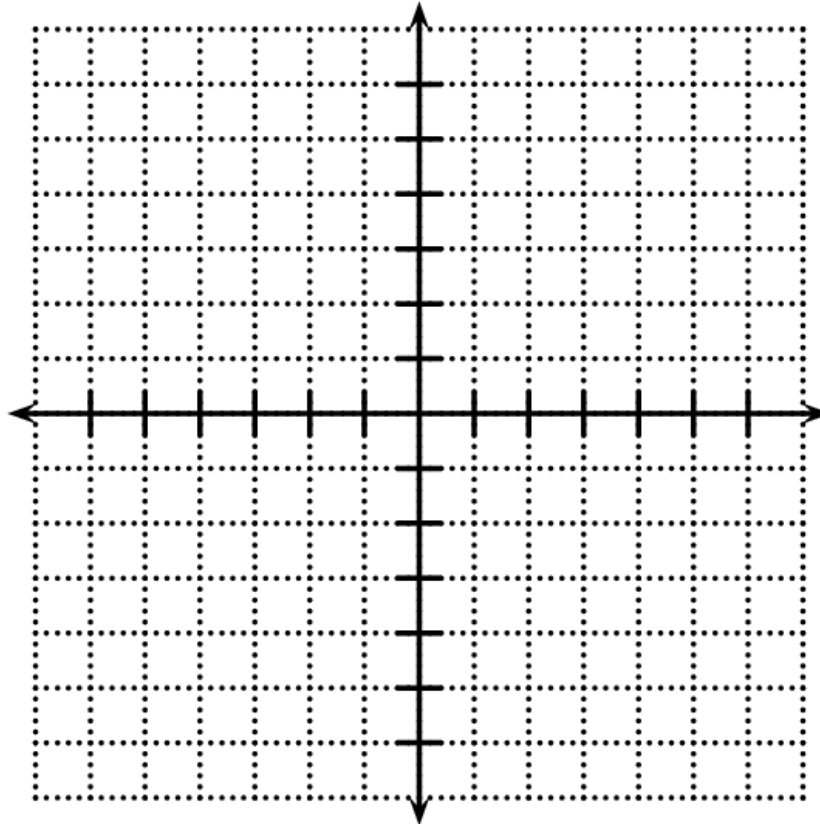
Let's first solve by graphing:



- The answer is the point  $(x, y)$  where the lines meet: \_\_\_\_\_
- Check the answer by plugging the point back into BOTH equations.

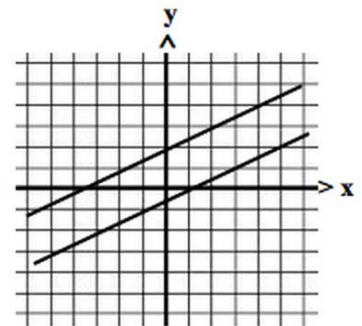
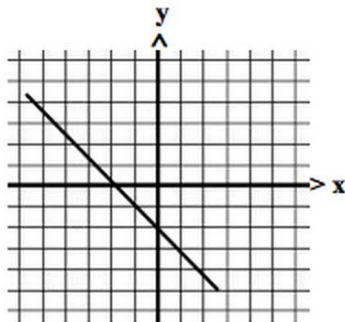
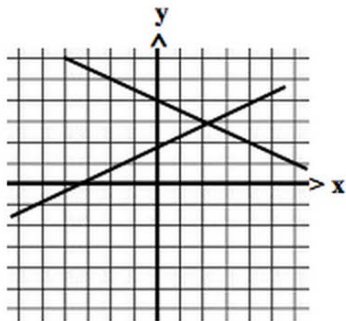
Let's find where these lines meet:

$$y = \frac{3}{2}x - 4$$
$$3x - 2y = 2$$



### Study Edge Tip

Two lines can either intersect once (one solution), all the time (infinitely many solutions), or not at all (no solution).



## BEAT THE TEST!

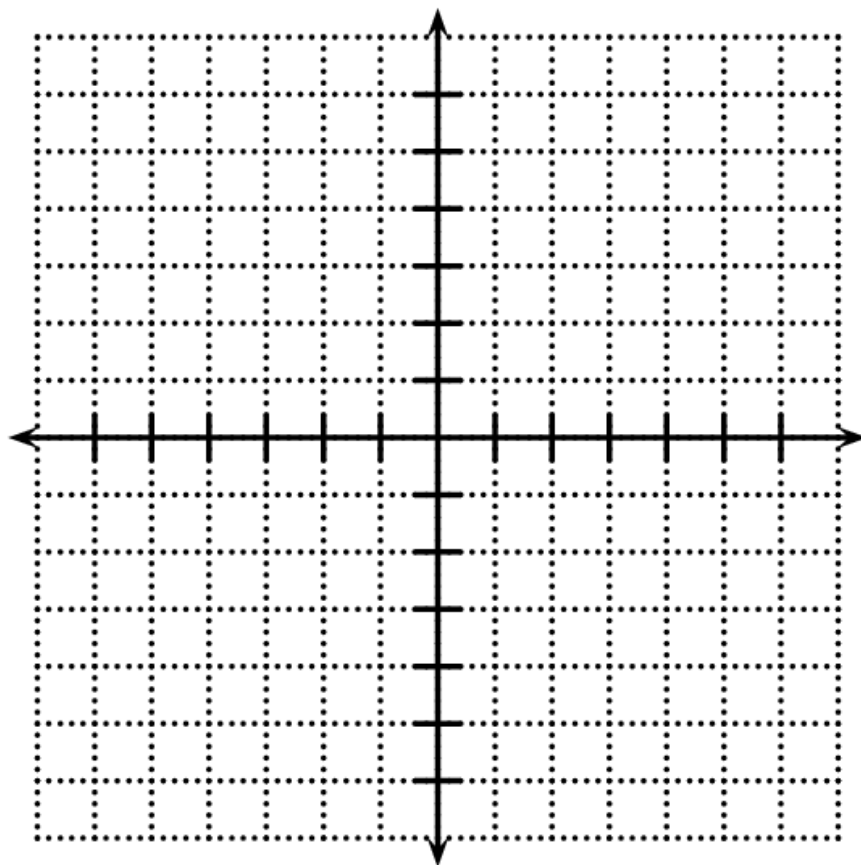
1. Given the following system of equations:

$$y = 2x - 6$$

$$x + y = -4$$

How many times did the lines intersect?

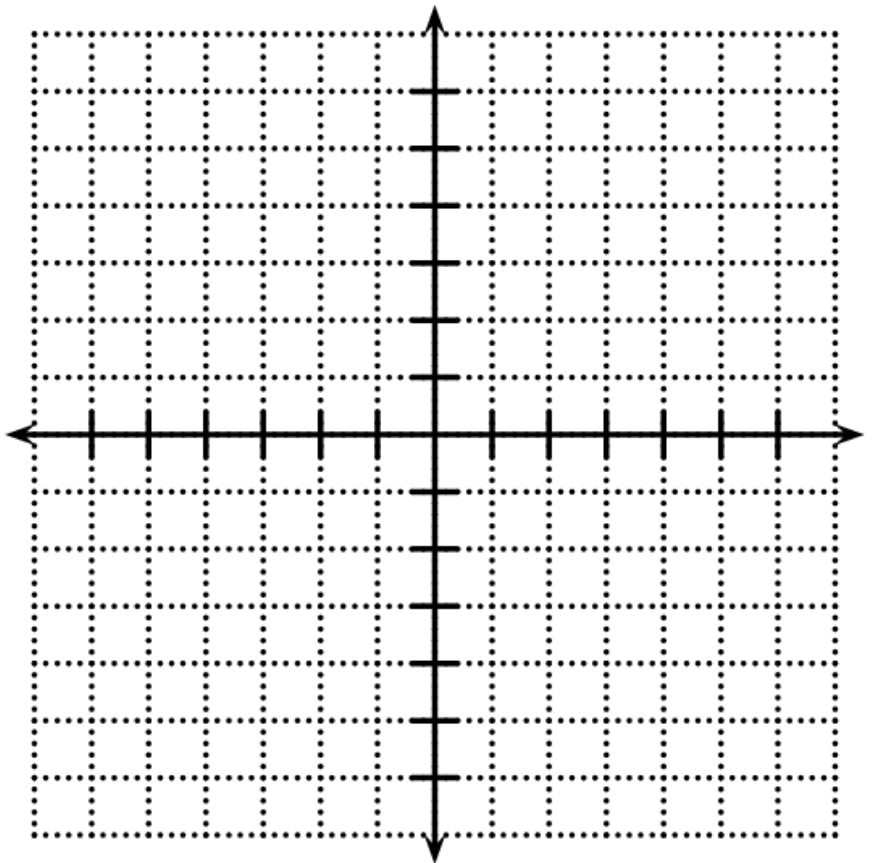
- A. None, the lines are parallel
- B. 1
- C. 2
- D. Infinitely many



2. Javier was walking along the path  $y = \frac{3}{2}x + 2$ . Damien was following a different path  $5x - y = 5$ .

At what point do the two boys meet?

- A. (0,0)
- B. (3,2)
- C. (2,5)
- D. No solution



## Section 7 – Video 2

### Solving Systems Using Algebra

- Graphing is a good way to see a solution, but it's not easy to see if there are fractions and decimals.
- We are going to talk about two other algebraic methods:
  - **Substitution**
  - **Elimination**
- It is important to know which one to use, so you can get done quickly (work smarter, not harder)!

<b>Substitution</b>	<b>Elimination</b>
<ul style="list-style-type: none"> <li>➤ Use when you can easily solve for a variable.</li> <li>➤ When a variable has a coefficient of 1.</li> </ul> $x + 3y = 7$ $2x - 5y = 6$ <p style="margin-top: 20px;">Which variable is easy to solve for?</p>	<p>Use when the same variable has the same coefficient.</p> $20x + 45y = 150$ $13x + 45y = 172$ <p style="margin-top: 20px;">Or, when one coefficient is a multiple of the other:</p> $3x + 2y = 10$ $6x + 7y = 15$

#### **Study Edge Tip**

On the test, you will not be told which method to use. Both work every time, but some problems are easier to solve with substitution and some are easier using elimination.

Let's start off with *just* identifying which method to use.

$$y = -2x + 7$$
$$11x + 9y = 2$$

- ELIMINATION
- SUBSTITUTION

*Why?*

$$8x + 6y = 3$$
$$11x + 6y = 2$$

- ELIMINATION
- SUBSTITUTION

*Why?*

$$4x + 3y = 11$$
$$8x + 8y = 9$$

- ELIMINATION
- SUBSTITUTION

*Why?*

To actually solve these, we'll break it down into three categories:

1. Substitution
2. Elimination Match
3. Elimination Multiple

## Section 7 – Video 3

### Solving a System of Equations by Substitution

$$2x + y = 4$$

$$3x + 4y = 1$$

$$-x + y = 1$$

$$2x + y = -2$$



*Try it!*

$$x = 7 - 4y$$

$$4x - 3y = 9$$

$$y = x - 4$$

$$4x + y = 26$$

# BEAT THE TEST!

1. Given the system of equations:

$$8c - 9t = -10$$

$$c + 8t = 90$$

What is the value of  $t$ ?

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## Section 7 – Video 4

### Solving a System of Equations by Elimination Match

$$-10x + 3y = 4$$

$$10x + 5y = -20$$

#### **Study Edge Tip**

If the numbers aren't opposites, change all the signs in one of the equations!

$$x - y = 0$$

$$-3x - y = 2$$

*Try it!*

$$\begin{aligned}2x + 5y &= -7 \\ -3x + 5y &= 3\end{aligned}$$

## BEAT THE TEST!

1. Given the system of equations:

$$8c - 9t = 10$$

$$2c - 9t = 70$$

What is the value of  $t$ ?

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2. Given the system of equations:

$$a - b = -5$$

$$a + 2b = 4$$

What is the value of  $b$ ?

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## Section 7 – Video 5

### Solving a System of Equations by Elimination Multiple

$$11x + 4y = 1$$

$$7x + 8y = 17$$

*Try it!*

$$7x - 6y = 4$$

$$-x + 12y = -34$$

## BEAT THE TEST!

1. Heidi is creating a clothing line. To make 7 jackets and 3 shirts will cost her \$17. However, if she wants to make 5 jackets and 12 shirts, it will cost \$22. Below is the system of equations representing her cost:

$$\begin{aligned}7j + 3s &= 17 \\5j + 12s &= 22\end{aligned}$$

What is the cost of making each jacket?

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## Section 7 – Video 6

### Solving Word Problems with Systems

When it comes to word problems, the tricky part is knowing what you're solving for.

- **If it is a multiple choice question (A, B, C, D):**
  - Solve the system; you can check your answers by plugging back into both equations.
  - Make sure you are solving for the correct variable.
- **If it is a free response question** :
  - You will only have to give the correct answer for one of the variables.
  - If you give the right answer for the wrong variable, your answer will be marked wrong.



For these examples, let's practice just writing the equations:

Carmen spent a total of \$19.50 at a concession stand on snacks for herself and her friends. Hamburgers cost \$3.50 and hot dogs cost \$2.25. If she bought a total of 7 snacks, how many hot dogs did she buy?

There are 13 animals in the barn. Some are chickens and some are pigs. There are 40 legs in all. How many of each animal are there?

## BEAT THE TEST!

1. The admission fee at a play is \$2.50 for each child and \$4.00 for each adult. On opening night, the 1000 seat auditorium was sold out! When all the money was counted, they had \$3400.

How many adults attended the opening night's show?

- A. 100
- B. 400
- C. 500
- D. 600

2. Anita Potter was selling meal tickets for the Criminal Justice program at her school. The cost of a dinner is \$10, and if you also wanted access to the dessert buffet, it is an additional \$12. She lost track of how many of each ticket she sold! However, she remembered that she was given 100 tickets and she collected \$1120.

If  $x$  is the number of dinner tickets and  $y$  is the number of dessert tickets, how many dessert tickets did she sell? The following equations model the situation:

$$\begin{aligned}x + y &= 100 \\10x + 12y &= 1120\end{aligned}$$

How many meals with desserts did she sell?

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3. Hugh Jackson wanted to add some new plants to his garden. He wanted to buy some rose bushes and grapevines. The first order was for 8 rose bushes and 4 grapevines and cost \$140. When he saw that there was more room, he ordered 5 more rose bushes and 5 more grapevines. The second order cost \$110. There was still more room for grapevines, but first he wanted to know how much each one costs.

Which system of equations could be used to find out the cost of each grapevine?

A.  $8g + 4r = 140$   
 $5g + 5r = 110$

B.  $8r + 4g = 110$   
 $5r + 5g = 140$

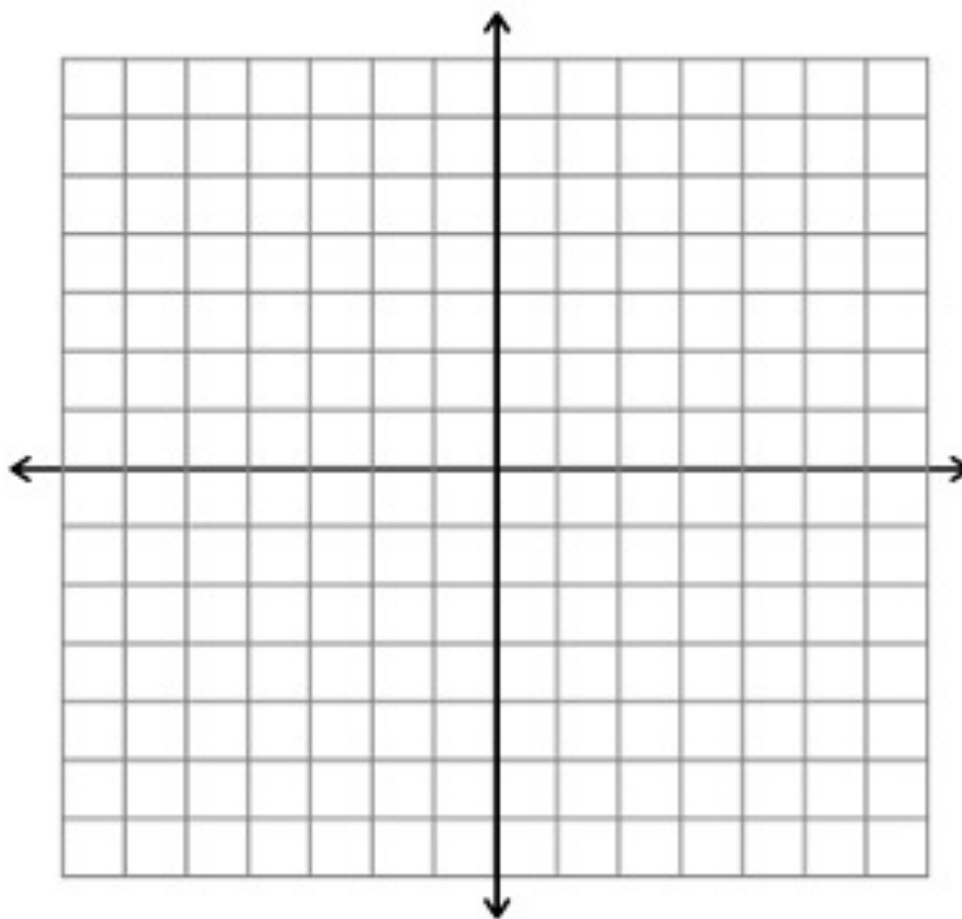
C.  $8g + 4r = 110$   
 $5g + 5r = 140$

D.  $8r + 4g = 140$   
 $5r + 5g = 110$

## Section 7: Systems of Equations

### Practice Problems

1. There are two limos heading downtown for a concert. One of the limos is driving Toucan Sam and the other limo is driving Tony the Tiger. Toucan Sam's limo is following the line  $x + y = 3$  and Tony the Tiger's limo is following the line  $-3x + y = -1$ . Graph each one of these lines on the plane below and label the point where the two routes intersect.



2. President Barack Obama and Vice President Joe Biden are having a competition on who can solve this system of equations the fastest!

$$\begin{aligned}2x + 5y &= 13 \\ x &= 5 + 3y\end{aligned}$$

Which method would you use to solve this system of equations?  
Why did you choose that method?

Now that you've decided which method will help you solve the system fastest, what is the solution to this system of equations?

3. Katie earned \$12,000 last year by giving tennis lessons. She invested part of her earnings at 3% simple interest and the rest at 4% simple interest. In one year, she earned \$440 in interest. How many dollars did she invest at 3% simple interest?

4. Leslie joins a fitness club that has an initial membership fee of \$20 plus \$15 per month. Rashad's gym has an initial fee of \$40 and charges \$10 per month. In how many months will the two fitness clubs cost the same? Whose gym is cheaper after 6 months?

5. Tiffany and Tammie are trying to determine when their paths will cross for a mathematical game. Tiffany travels down the path defined by the equation  $4x - 2y = 16$  and Tammie travels down the path defined by the equation  $4x + 4y = 28$ . What is the point at which these two paths cross? What method did you use to solve the system and why?
6. A farmer mixes cashews that cost \$2.49 per pound and almonds that cost \$3.89 per pound to make 100 pounds of a nut mixture that costs \$3.26 per pound. How many pounds of cashews were put into the mixture?



7. For a show on Broadway, admission is \$200.00 for an adult and \$50.00 a child. The theater has a maximum capacity of 500 and was sold out for the matinee showing of a popular show. The total amount collected at the box office in admission fees for that showing was \$79,750. How many adults attended the matinee showing?
8. John has \$4 and plans to spend all of it on candy. The cost of each Hershey Kiss is 20 cents and the cost of a Blow Pop is 40 cents. If John buys a total of 16 pieces of candy and only buys Hershey Kisses and Blow Pops, how many of each candy type did he buy?

9. A test has twenty questions worth 100 points total. The test consists of True/False questions, each worth 3 points, and multiple-choice questions, each worth 11 points. How many multiple-choice questions are on the test?
10. Samantha moved out of her house and sold some tables and chairs. She sold 30 chairs and tables and made a total of \$1,640. If each chair sold for \$50 and each table sold for \$60, how many chairs and how many tables did Samantha sell?