



Mathematics for College Readiness (#1200700)

This document was generated on CPALMS - www.cpalms.org

Course Number: 1200700

Course Section: Grades PreK to 12 Education Courses

Number of Credits: One credit (1)

Course Type: Core Course

Course Status: Course Approved

Keywords: PreK to 12 Education, Pre K to 12 Education, Grades 9 to 12 and Adult Education, 9 to 12, 9-12, High School, Mathematics, Math, Algebra, Mathematics for College Readiness, MATH COLL READINESS, College Readiness

Grade Level(s): 9, 10, 11, 12

Course Path: Section: Grades PreK to 12 Education Courses > **Grade Group:** Grades 9 to 12 and Adult Education Courses > **Subject:** Mathematics > **SubSubject:** Algebra >

Abbreviated Title: MATH COLL READINESS

Course Length: Year (Y)

Course Level: 2

GENERAL NOTES

This course is targeted for students who are not yet "college ready" in mathematics or simply need some additional instruction in content to prepare them for success in college level mathematics. This course incorporates the Florida Standards for Mathematical Practices as well as the following Florida Standards for Mathematical Content: Expressions and Equations, The Number System, Functions, Algebra, Geometry, Number and Quantity, Statistics and Probability, and the Florida Standards for High School Modeling. The standards align with the Mathematics Postsecondary Readiness Competencies deemed necessary for entry-level college courses.

English Language Development ELD Standards Special Notes Section:

Teachers are required to provide listening, speaking, reading and writing instruction that allows English language learners (ELL) to communicate information, ideas and concepts for academic success in the content area of Mathematics. For the given level of English language proficiency and with visual, graphic, or interactive support, students will interact with grade level words, expressions, sentences and discourse to process or produce language necessary for academic success. The ELD standard should specify a relevant content area concept or topic of study chosen by curriculum developers and teachers which maximizes an ELL's need for communication and social skills. To access an ELL supporting document which delineates performance definitions and descriptors, please click on the following link:

<http://www.cpalms.org/uploads/docs/standards/eld/MA.pdf>

For additional information on the development and implementation of the ELD standards, please contact the Bureau of Student Achievement through Language Acquisition at sala@fldoe.org.

Additional Instructional Resources:

A.V.E. for Success Collection: <http://www.fasa.net/iTunesU/index.cfm>

Course Standards

Name	Description
ELD.K12.ELL.MA.1:	English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.
ELD.K12.ELL.SI.1:	English language learners communicate for social and instructional purposes within the school setting.
	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. <ol style="list-style-type: none"> Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.
MAFS.7.EE.2.4:	<p>Remarks/Examples:</p> <p>Fluency Expectations or Examples of Culminating Standards</p> <p>In solving word problems leading to one-variable equations of the form $px + q = r$ and $p(x + q) = r$, students solve the equations fluently. This will require fluency with rational number arithmetic (7.NS.1.1–1.3), as well as fluency to some extent with applying properties operations to rewrite linear expressions with rational coefficients (7.EE.1.1).</p> <p>Examples of Opportunities for In-Depth Focus</p> <p>Work toward meeting this standard builds on the work that led to meeting 6.EE.2.7 and prepares students for the work that will lead to meeting</p>

8.EE.3.7.

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

- a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
- b. Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- d. Apply properties of operations as strategies to add and subtract rational numbers.

MAFS.7.NS.1.1:

Remarks/Examples:

Fluency Expectations or Examples of Culminating Standards

Adding, subtracting, multiplying, and dividing rational numbers is the culmination of numerical work with the four basic operations. The number system will continue to develop in grade 8, expanding to become the real numbers by the introduction of irrational numbers, and will develop further in high school, expanding to become the complex numbers with the introduction of imaginary numbers. Because there are no specific standards for rational number arithmetic in later grades and because so much other work in grade 7 depends on rational number arithmetic, fluency with rational number arithmetic should be the goal in grade 7.

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

- a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
- c. Apply properties of operations as strategies to multiply and divide rational numbers.
- d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

MAFS.7.NS.1.2:

Remarks/Examples:

Fluency Expectations or Examples of Culminating Standards

Adding, subtracting, multiplying, and dividing rational numbers is the culmination of numerical work with the four basic operations. The number system will continue to develop in grade 8, expanding to become the real numbers by the introduction of irrational numbers, and will develop further in high school, expanding to become the complex numbers with the introduction of imaginary numbers. Because there are no specific standards for rational number arithmetic in later grades and because so much other work in grade 7 depends on rational number arithmetic, fluency with rational number arithmetic should be the goal in grade 7.

MAFS.8.EE.1.1:

Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*

MAFS.8.EE.1.4:

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for sea floor spreading). Interpret scientific notation that has been generated by technology.

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

MAFS.8.EE.2.5:

Remarks/Examples:

Examples of Opportunities for In-Depth Focus

When students work toward meeting this standard, they build on grades 6–7 work with proportions and position themselves for grade 8 work with functions and the equation of a line.

MAFS.8.F.2.4:

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MAFS.8.NS.1.1:

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

MAFS.8.NS.1.2:

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., n^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

MAFS.912.A-APR.1.1:

Remarks/Examples:

Algebra 1 - Fluency Recommendations

Fluency in adding, subtracting, and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

MAFS.912.A-APR.2.3:

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

MAFS.912.A-APR.3.4:

Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

MAFS.912.A-APR.4.6:	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
MAFS.912.A-APR.4.7:	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MAFS.912.A-CED.1.1:	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational, absolute, and exponential functions. ★
MAFS.912.A-CED.1.2:	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
MAFS.912.A-CED.1.3:	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. ★
MAFS.912.A-CED.1.4:	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law $V = IR$ to highlight resistance R.</i> ★
MAFS.912.A-REI.1.1:	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MAFS.912.A-REI.1.2:	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
MAFS.912.A-REI.2.3:	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
MAFS.912.A-REI.2.4:	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a ± bi$ for real numbers a and b .
MAFS.912.A-REI.3.5:	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
MAFS.912.A-REI.3.6:	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
MAFS.912.A-REI.4.10:	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
MAFS.912.A-REI.4.11:	Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★
MAFS.912.A-SSE.1.1:	Interpret expressions that represent a quantity in terms of its context. ★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P .
MAFS.912.A-SSE.1.2:	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MAFS.912.A-SSE.2.3:	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} ≈ 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
MAFS.912.F-BF.1.1:	Write a function that describes a relationship between two quantities. ★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. c. Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
MAFS.912.F-BF.2.3:	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
MAFS.912.F-IF.1.1:	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
MAFS.912.F-IF.2.4:	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★
MAFS.912.F-IF.2.5:	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble engines in a factory, then the positive integers would be an appropriate domain for the function. ★
MAFS.912.F-IF.2.6:	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★
MAFS.912.F-IF.3.7:	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

- d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude, and using phase shift.

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

[MAFS.912.F-IF.3.8:](#)

- a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$; $y = (0.97)^t$; $y = (1.01)^{12t}$; $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

[MAFS.912.G-GPE.2.5:](#)

Remarks/Examples:
Geometry - Fluency Recommendations

Fluency with the use of coordinates to establish geometric results, calculate length and angle, and use geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields.

[MAFS.912.G-GPE.2.6:](#)

Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★

[MAFS.912.G-GPE.2.7:](#)

Remarks/Examples:
Geometry - Fluency Recommendations

Fluency with the use of coordinates to establish geometric results, calculate length and angle, and use geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields.

[MAFS.912.N-Q.1.1:](#)

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★

Define appropriate quantities for the purpose of descriptive modeling. ★

[MAFS.912.N-Q.1.2:](#)

Remarks/Examples:
Algebra 1 Content Notes:

Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.

[MAFS.912.N-Q.1.3:](#)

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★

[MAFS.912.N-RN.1.1:](#)

Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3) \cdot 3}$ to hold, so $(5^{1/3})^3$ must equal 5.

[MAFS.912.N-RN.1.2:](#)

Rewrite expressions involving radicals and rational exponents using the properties of exponents.

[MAFS.912.N-RN.2.3:](#)

Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

[MAFS.912.S-ID.2.5:](#)

Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★

[MAFS.912.S-ID.2.6:](#)

- a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, and exponential models.
- b. Informally assess the fit of a function by plotting and analyzing residuals.
- c. Fit a linear function for a scatter plot that suggests a linear association.

Remarks/Examples:

Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.

[MAFS.912.S-ID.3.7:](#)

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★

Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

[MAFS.K12.MP.1.1:](#)

Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of

[MAFS.K12.MP.2.1:](#)

quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

[MAFS.K12.MP.3.1:](#)

Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

[MAFS.K12.MP.4.1:](#)

Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

[MAFS.K12.MP.5.1:](#)

Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

[MAFS.K12.MP.6.1:](#)

Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

[MAFS.K12.MP.7.1:](#)

Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

[MAFS.K12.MP.8.1:](#)

There are more than 905 related instructional/educational resources available for this on CPALMS. Click on the following link to access them: <http://www.cpalms.org/Public/PreviewCourse/Preview/13044>

Related Certifications

[Mathematics Grades 6-12](#)