Name: _____

Skill Sheet 7.1C — Pythagorean Theorem

When you know the *x*- and *y*- components of a vector, you can find its magnitude using the Pythagorean theorem. This useful theorem states that $a^2 + b^2 = c^2$, where *a, b,* and *c* are the lengths of the sides of any right triangle. For example, suppose you need to know the distance represented by the displacement vector (4,3)m. If you walked east 4 meters then north 3 meters, you would walk a total of 7 meters. This is a distance, but it is not the distance specified by the vector, or the shortest way to go. The vector (4,3)m describes a single straight line. The length of the line is 5 meters because $4^2 + 3^2 = 5^2$.

The Pythagorean theorem can be used to help us calculate the magnitude of a vector once we know its components along the *x*- and *y*- directions. Also, we can find one of the components of the vector if we know the other component and the magnitude of the vector.

1. Example problem

A displacement vector $\vec{x} = (2,3)\text{m}$ has these components:

2 meters in the *x* direction.

3 meters in the *y* direction.

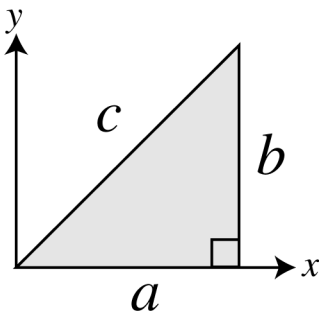
What is the magnitude of the vector?

The Pythagorean theorem

$$a^2 + b^2 = c^2$$

a and b are the lengths of the short sides of a right triangle.

c is the length of the side opposite the right angle.



Using the Pythagorean theorem, *a* is the component along the *x* direction and *b* is the component along the *y* direction. The magnitude of the vector is *c*. We can find the magnitude by taking the square root of $a^2 + b^2$:

$$\sqrt{a^2 + b^2} = \sqrt{c^2}$$

$$\sqrt{(2\text{ m})^2 + (3\text{ m})^2} = \sqrt{c^2}$$

$$\sqrt{4\text{ m}^2 + 9\text{ m}^2} = \sqrt{c^2}$$

$$\sqrt{13\text{ m}^2} = 3.6\text{ m} = c$$

2. Solving problems

1. Find the magnitude of the vector $\vec{a} = (3, 4)$.

2. Find the magnitude of the vector $\vec{v} = (-3, -4)$.

3. Find the magnitude of the vector $\vec{v} = (5, 0)$.

4. Find the magnitude of the vector $\vec{x} = (12.00, 6.00)$cm.

5. A robot starts from a certain point and moves east for a distance of 5.0 meters, then goes north for 3.0 meters, and then turns west for 2.0 meters.
   a. What are the *x*-*y* coordinates for the resultant vector?

   b. What is the magnitude of the resultant vector for the robot?

6. Add the vectors $\vec{v}_1 = (5,0)$, $\vec{v}_2 = (0,–3)$, and $\vec{v}_3 = (1,0)$, and find the magnitude of the resultant vector.

7. Add the vectors $\vec{v}_1 = (–5,0)$, $\vec{v}_2 = (0,–2)$, and $\vec{v}_3 = (7,0)$, and find the magnitude of the resultant vector.

8. A resultant vector has a magnitude of 25 meters. Its *y* component is –12 meters. What is its *x* component?

3. Challenge problems

1. Express the resultant vector in problem 5, Part 2 in polar coordinates. Assume that the positive *x* direction is from west to east and the positive *y* direction is from south to north.

2. Add the vectors $\vec{v}_1 = (5,0)$, $\vec{v}_2 = (0,–5)$, and $\vec{v}_3 = (5,180°)$, and find the magnitude of the resultant vector.

3. Add the vectors $\vec{v}_1 = (5,45°)$, $\vec{v}_2 = (0,–10)$, and $\vec{v}_3 = (1,180°)$, and find the magnitude of the resultant vector.