Name:

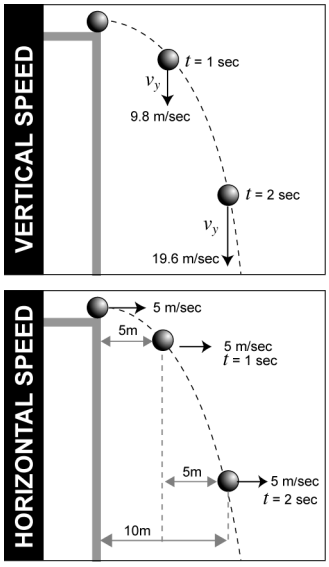
Skill Sheet 7.2 Projectile Motion

This is a review of projectile motion. The problems in the skill sheet will give you practice in solving problems that involve projectile motion.

1. Projectile motion has vertical and horizontal components

Projectile motion has vertical and horizontal components. Gravity affects the vertical motion of an object. When we drop a ball from a height, we know that its speed increases as it falls. The increase in vertical speed is due to the acceleration gravity, $g = 9.8$ m/sec^2. So the vertical speed of the ball will increase by 9.8 m/sec after each second. After the first second has passed, the speed will be 9.8 m/sec. After the next second has passed, the speed will be 19.6 m/sec and so on.

The acceleration of gravity affects only the vertical component of the motion. Horizontal motion is not affected by gravity. So if we neglect the friction from air, when we throw an object horizontally, its initial horizontal speed will not change. For example, if we throw a marble horizontally at a speed of 5 m/sec, the marble will be 5 meters horizontally from our hand after one second, 10 meters after 2 seconds, and so forth.



2. Solving projectile motion problems

Solving projectile motion problems requires using equations. To solve these problems, follow the steps:

- Read the problem carefully. You may want to diagram the problem to help you understand it.
- List what you know from the problem and what you need to solve for.
- Determine which equations for vertical motion or horizontal motion will help you solve the problem. You may need more than one equation to solve the problem. Some important equations are listed below.
- Solve the problem and check your work.

Pythagorean theorem	$a^2 + b^2 = c^2$	Use this equation to find the magnitude of a velocity vector, (a, b).
Horizontal motion	$v_{ox} = v_0 \cos\theta$	Use this equation to calculate initial horizontal velocity when you know an angle and magnitude of the initial velocity vector.
Vertical motion	$v_{oy} = v_0 \sin\theta$	Use this equation to calculate initial vertical velocity when you know an angle and magnitude of the initial velocity vector.
Horizontal distance	$x = v_{ox}t$	This equation is a rearranged version of the speed equation: v = d/t. Here, x represents d, distance.
Vertical velocity	$v_y = v_{oy} - gt$	Gravity (g) is included in these equations because vertical speed accelerates due to gravity when an object is falling.
Vertical distance	$y = v_{oy}t - \frac{1}{2}gt^2$	
The time to reach maximum height	$t = \frac{v_{oy}}{g}$	This equation is a rearranged version of acceleration = speed/time.

3. Sample projectile motion problem

A ball is kicked with an initial total velocity (v_0) of 10 m/sec at an angle of 60 degrees off the ground. The time that it takes for the ball to reach the ground again is twice the time it takes for the ball to reach its maximum height. Using this information, estimate how far the ball will go horizontally and the maximum height it will reach.

The horizontal (or x) component of the ball's velocity is:

$v_{ox} = v_0\cos(60°) = 10 \text{ m/sec} \times 0.5 = 5 \text{ m/sec}$

The vertical (or y) component of the ball's velocity is:

$v_{oy} = v_0\sin(60°) = 10 \text{ m/sec} \times 0.87 = 8.7 \text{ m/sec}$

The time it takes for the ball to reach its maximum height (t) is written below. The initial vertical velocity is v_{oy} and g is the acceleration due to gravity.

$$t = \frac{v_{oy}}{g}$$

The total time it takes for the ball to travel horizontally is twice this long:

$$t = 2 \times \left(\frac{v_{oy}}{g}\right) = 2 \times \left(\frac{8.7 \text{ m/sec}}{9.8 \text{ m/sec/sec}}\right) = 1.8 \text{ sec}$$

With this information, we are now able to answer the questions:

- What is the horizontal range of the ball?
- What is the maximum height reached by the ball?

The horizontal range equals the speed times the time in the horizontal direction:
$x = v_{ox} \times t = 5 \text{ m/sec} \times 1.8 \text{ sec} = 9 \text{ m}$

The maximum height—or vertical distance (y)—can be calculated using the formula below.

$$y = [v_{oy}t] - \left[\frac{1}{2} gt^2\right]$$

In this problem, the ball reaches its maximum height in half the time that the ball travels before reaching the ground. This time has been calculated to be 1.8 seconds. Therefore, 1/2 times 1.8 seconds is used in the equation below for vertical distance.

$$y = \left[v_{oy} \times \frac{1}{2}(\text{time to travel horizontally})\right] - \left[\frac{1}{2} g \times \frac{1}{2}(\text{time to travel horizontally})^2\right]$$

$$y = 8.7 \text{ m/sec} \times \frac{1.8}{2} \text{ sec} - \frac{1}{2} 9.8 \text{ m/sec}^2 \left[\frac{1.8}{2} \text{ sec}\right]^2 = 3.82 \text{ m}$$

4. Solving problems

Solve the following problems. Show your work.

1. A cat runs and jumps from one roof top to another which is 5 meters away and 3 meters below. Calculate the minimum horizontal speed with which the cat must jump off the first roof in order to make it to the other.
   a. What do you know?

   b. What do you need to solve for?

   c. What equation (s) will you use?

   d. What is the solution to this problem?

2. An object is thrown off a cliff with a horizontal speed of 10 m/sec and some unknown initial vertical velocity. After 3 seconds the object hits the ground which is 30 meters below the cliff. Find the initial vertical velocity and the total horizontal distance traveled by the object.
   a. What do you know?

   b. What do you need to solve for?

   c. What equation (s) will you use?

   d. What is the solution to this problem?

3. If a marble is released from a height of 10 meters how long would it take for it to hit the ground?

4. A ball is thrown vertically upwards with a speed of 5 m/sec. How long before the ball hits the ground? (HINT: Consider that there will be time for the ball to go up and then fall back down.)

5. A ski jumper competing for an Olympic gold medal wants to jump a horizontal distance of 135 meters. The takeoff point of the ski jump is at a height of 25 meters. With what horizontal speed must he leave the jump?

6. An object is launched at an angle of 45 degrees with a speed of 20 m/sec. Calculate the initial velocity components, the time it takes to hit the ground, the range, and the maximum height it reaches.