Name: ______________________________

Skill Sheet 17.3 Thin Lens Formula

Here you will become familiar and practice with a mathematical formula called “the thin lens formula.” This formula gives scientists a way to calculate the location and the size of an image that is produced by a lens.

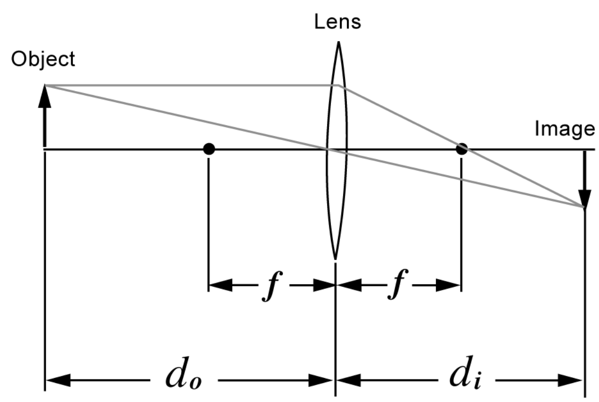
1. What is the thin lens formula?

When you use the thin lens formula, you assume that the thickness of the lens is very small compared with the distance between the lens and the object or the image. The formula applies both to convex or converging lenses and concave or diverging lenses. Converging lenses are thicker in the center than in the edges. Diverging lenses are thinner in the center than in the edges. The thin lens formula is:

Thin lens formula

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Object distance (cm) | Image distance (cm) | Focal length (cm)

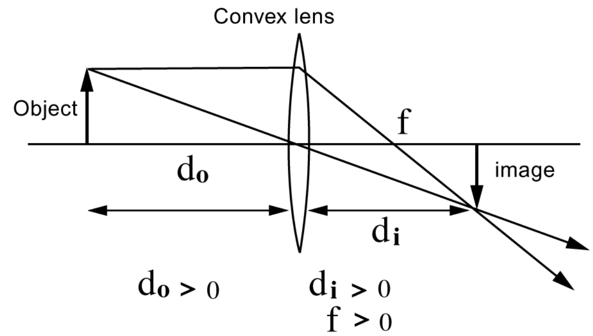


Some important rules in using the thin lens formula:

- Object distance, d_0, is positive to the left of the lens and negative to the right of the lens.
- Image distance, d_i, is negative to the left of the lens and positive to the right of the lens.
- Positive d_0, d_i indicates real object or image.
- Negative d_0, d_i indicates virtual object or image.

2. Examples in using the thin lens formula

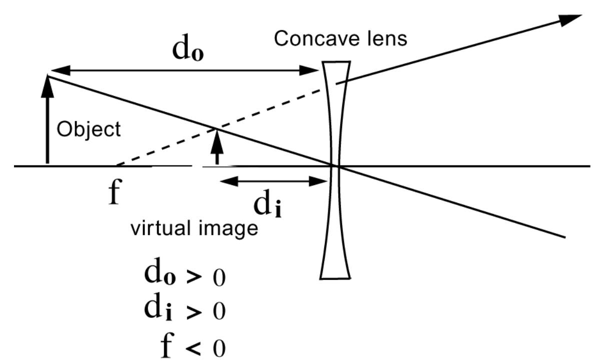
If you are using a **convex lens**, what happens an image when the object is very far from the lens? In other words, what happens when d_0 is a large number?



Graphically, we see that as d_0 increases, the image becomes smaller as it gets closer to the focal point. We can see this by using the thin lens formula with a lens that has a focal length equal to 5 centimeters.

If d_0 =	then...	d_i =
8 cm,	$\frac{1}{d_i} = \frac{1}{5} - \frac{1}{8} = 0.2 - 0.125 = 0.075$	$\frac{1}{0.075} = 13.3$ cm
100 cm,	$\frac{1}{d_i} = \frac{1}{5} - \frac{1}{100} = 0.2 - 0.01 = 0.19$	$\frac{1}{0.19} = 5.26$ cm
1,000 cm,	$\frac{1}{d_i} = \frac{1}{5} - \frac{1}{1{,}000} = 0.2 - 0.001 = 0.199$	$\frac{1}{0.199} = 5.03$ cm

If you are using a **concave lens**, you use a negative value for the focal length, *f*. In this case, the resulting image distance is a negative number indicating a virtual image. The calculations are shown below:



If d_0 =	then...	d_i =
8 cm,	$\frac{1}{d_i} = -\frac{1}{5} - \frac{1}{8} = -0.2 - 0.125 = -0.325$	$\frac{1}{-0.325} = -4.71$ cm
100 cm,	$\frac{1}{d_i} = -\frac{1}{5} - \frac{1}{100} = -0.2 - 0.01 = -0.21$	$\frac{1}{-0.21} = 4.76$ cm
1,000 cm	$\frac{1}{d_i} = -\frac{1}{5} - \frac{1}{1{,}000} = -0.2 - 0.001 = -0.201$	$\frac{1}{-0.201} = -4.98$ cm

3. Problems

1. Calculate the location of the image if the object is 20 centimeters in front of a convex (converging) lens with a focal length of 5 centimeters.

2. The image of an object as seen by a converging lens is located at 8 centimeters. The object is 24 centimeters in front of the lens. What is the focal length?

3. For a converging lens with a focal length of 10 centimeters, calculate the location of the object when the image appears 20 centimeters to the right.

4. An object is 10 centimeters in front of a convex (converging) lens with a focal length of 15 centimeters.
   a. Calculate the location of the image.

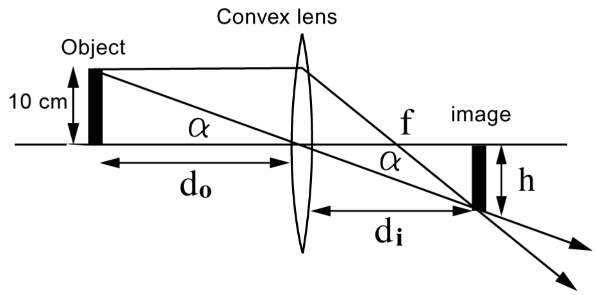
   b. What kind of image appears in this situation? Where does it appear in relation to the lens?

5. Calculate the location of the image if the object is 20 centimeters in front of a concave (diverging) lens with a focal length of 5 centimeters.

6. An object is at a distance of 3 centimeters from a lens with a focal length of 1 centimeter. The lens creates an image on the same side of the object. What kind of lens is this? What is the image location? Is the image real or virtual?

4. Problems that involve the height of images and objects

Sample problem: The object is 10 centimeters in height and is located at a distance of 25 centimeters from the lens. The focal length is 8 centimeters. Find the image location d_i and the height h of the image.



Solution:

Since we know the height of the object and the object distance d_0, we can calculate the angle α.

$$\alpha = \tan^{-1}\left(\frac{10}{25}\right) = \tan^{-1}(0.4) = 21.8°$$

The image distance d_i can be found from the thin lens formula, and it is 11.7 centimeters.

Now, since we know the image distance and the angle α, we can calculate the height.

$$h = \tan\alpha d_i = \tan 21.8° 11.17\text{ cm} = 4.7\text{ cm}$$

So we see that the image is actually smaller than the object.

1. Prove that the d_i in the problem above is 11.7 centimeters by using the thin lens formula.

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2. A 14-centimeter tall object seen through a lens with a focal length of 10 centimeters has an image half its size.
   a. Calculate the location of the image and the object. (HINT: Use proportions.)

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   b. Using your answers for 2(a) and the height of the object, find the angle α.
